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tate facile perspecturus foret; jam vero, quoniam egregium illud Rei Medicæ Lumen amisimus, eadem aliis Eruditis perpendenda simul proponimus & dijudicanda. Tibi præsertim, Vir Doctissime, cujus auctoritatem & ille plurimi fecit, & nos præcipuam habemus, Judici simul integerrimo & maxime idoneo, totam istam disputationem lubentissime subjicimus.

III. *Methodus Differentialis Newtoniana Illustrata.* *Authore Jacobo Stirling, è Coll. Balliol. Oxon.*

A Rithmeticæ pars præcipua consistit in inveniendâ in numeris quantitate quâcunque determinatâ; cum vero quantitatum & numerorum natura non patiatur ut omnes quantitates exhibeantur in numeris accurate, necesse habemus ad Approximationes confugere. Hoc est, ubi quantitatum valores mathematice accurati nequeunt obtineri, quærendi sunt ii qui ab accuratis distant minus datâ quâvis differentiâ.

Quicquid hâc de re à Veteribus ad nos pervenit, vel est particulare, ut Methodus eorum reducendi *Æquationes Quadraticas*; vel saltem usibus generalibus male destinatum, ut Methodus Exhaustionum. *Vieta* quidem primus erat qui aliquid generale in hâc arte assequutus est: quippe invenit methodum reducendi *Æquationes Rationales*, quæ solæ tunc in usu erant. In hâc acquievêre omnes Geometræ ex ejus temporibus usque ad ea *Newtoni*. Hic ex Interpolationibus primo pervenit ad Series: quas postea ad reductionem *Æquationum* omnium omnino generum universaliter applicuit. Hæc autem methodus procedit per quantitatum nascentium & evanescentium rationes primas & ultimas, seu si ita loqui liceat, per quantitatum coincidentium

cidentium differentias infinite parvas. Sed & ulterius promovit *Newtonus* hanc methodum; docuitque qua ratione approximandum sit ad quantitates quæ determinantur per regularem seriem terminorum, non per *Æquationem* ut vulgo fit. Atque sic posuit fundamenta calculi hujus Differentialis, qui procedit per quantitatum differentias cujuscunque magnitudinis: ideoque est methodo Serierum universalior. Per hæc artes *Newtonianas*, universa doctrina Approximationum reducitur ad solutionem Problematis, *Invenire Lineam Geometricam quæ per data quocunque puncta transibit*. Ex hujus inquam solutione inveniuntur radices *Æquationum* quarumcunque, & etiam quantitates quarum relationes ad alias datas per nullas *Æquationes* hæcenus notas possunt exprimi. Existimo igitur *Newtonum* perduxisse methodum Approximandi ad summum perfectionis fastigium; dum ex unico simplicissimo principio totam hanc doctrinam longe lateque patentem deducit. Quapropter credendum est animum *Newtoni* non satis perspectum fuisse iis, qui ejus methodos appellant particulares, & alias tanquam suas & solas genuinas atque generales venditant, quæ aliæ non erant quam Corollaria facillima à *Newtonianis*.

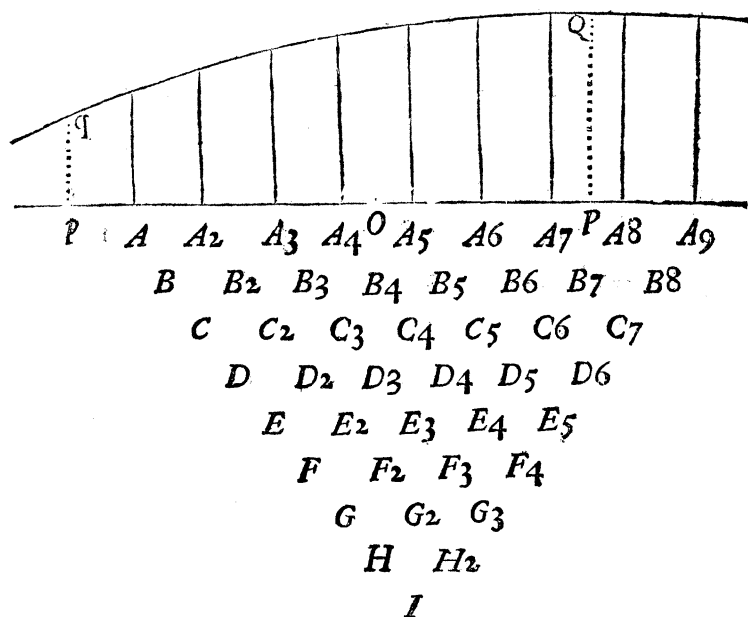
Author noster, in Epistola ad Oldenburgum, Octob. 24. 1676. data, mentionem fecit de methodo expeditâ ducendi Lineam Parabolicam per data quocunque puncta; qua dixit se usum fuisse ubi Series simplices non sunt satis tractabiles. Et hanc methodum primo publicavit in Lemmate quinto Libri tertii *Principiorum*. Atque in Lectionibus publicis, circa idem tempus quo dicta Epistola scripta est, *Cantabrigiæ* habitis, exposuit modum generalem determinandi Curvas cujuscunque generis quæ transibunt per totidem data puncta quor earum natura patitur. Hæ Lectiones sub titulo *Arithmetica Universalis* anno 1707. publicatæ sunt, ubi habetur

betur methodus exemplis illustrata in sectionibus Conicis. Anno vero 1711. tandem prodiit, inter alios ejusdem Authoris tractatus, ipsa Methodus Differentialis plenius quam ante exposita, cum fundamento ejus demonstrato

Archimedes in methodo Exhaustionum, *Cavallerius* in methodo Indivisibilium, & *Wallisus* noster in Arithmetica Infinitorum, posuerunt fundamenta doctrinæ de determinanda quantitate quæsitâ per locum quem obtinet inter terminos in data Serie: at qua ratione approximandum esset ad valores quantitatum sic determinatarum. horum nemo docuit; Hoc primus & solus perfecit *Newtonus*: atque exinde haud parum ampliata est universa Analysis. Nam sicut ante hoc inventum, ea Problemata Arithmetica sola pro solutis habebantur, ubi relatio quantitatis quæsitæ ad alias datas definiebatur *Æquatione*. jam pro solutis habenda sunt non minus ea, in quibus quantitas quæsitâ locum datum sortitur inter terminos datæ Seriei; siquidem numeri desiderati non minus accurate obtinentur per Methodum Differentialem, quam per extractionem Radicum: hisce vero habitis, parum interest quomodo ad eos devenitum est. Et experientia multiplex docuit, quod plurima Problemata ad *Æquationes* ægre deducuntur, dum ad methodum Differentialem facillime. Qualis est ex multis aliis toties decantata Circuli Quadratura; quam tam perfectam, mea opinione, *Wallisus* in Arithmetica Infinitorum exhibuit quam *Archimedes* illam Parabolæ.

Propositio.

Invenire Lineam Parabolicam quæ transibit per extrema Ordinatorum quotcumque æquidistantium.



Casus Primus.

Designent $A, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$, &c. Ordinatæ æquidistantes insistentes Abcissæ in dato angulo. Collige earum differentias $B, B_2, B_3, B_4, B_5, B_6, B_7, B_8$, &c. harumque differentias $C, C_2, C_3, C_4, C_5, C_6, C_7$, &c. harumque differentias $D, D_2, D_3, D_4, D_5, D_6$, &c. harumque differentias E, E_2, E_3, E_4, E_5 , &c. harumque F, F_2, F_3, F_4 , &c. Et sic porro. Differentiæ autem colligi debent auferendo

ferendo priores semper de posterioribus. Hoc est ponendo $B = A_2 - A$, $B_2 = A_3 - A_2$, $B_3 = A_4 - A_3$, $B_4 = A_5 - A_4$, $B_5 = A_6 - A_5$, &c. Tum $C = B_2 - B$, $C_2 = B_3 - B_2$, $C_3 = B_4 - B_3$, $C_4 = B_5 - B_4$, &c. deinde $D = C_2 - C$, $D_2 = C_3 - C_2$, $D_3 = C_4 - C_3$, &c. Et similiter sunt omnes differentiae sequentes colligendae. Vel sint $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta$, &c. æquales $A, A_2, A_3, A_4, A_5, A_6, A_7$, &c. Eritque $A = \alpha$, $B = \beta - \alpha$, $C = \gamma - 2\beta + \alpha$, $D = \delta - 3\gamma + 3\beta - \alpha$, $E = \epsilon - 4\delta + 6\gamma - 4\beta + \alpha$, $F = \zeta - 5\epsilon + 10\delta - 10\gamma + 5\beta - \alpha$, $G = \eta - 6\zeta + 15\epsilon - 20\delta + 15\gamma - 6\beta + \alpha$, &c. In hisce valoribus numerales Coefficientes ipsorum $\alpha, \beta, \gamma, \delta, \epsilon$ &c. generantur ut in dignitatibus integris Binomii $1 - z$, $1 - z^1$, $1 - z^2$, $1 - z^3$, $1 - z^4$, &c. Scribendo numeros 1, 2, 3, 4, 5, &c. in Serie $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. successive pro n . Sit jam P quælibet Ordinata reliquis intermedia, & AP ejus distantia ab Ordinata prima A appelletur z , tum erit

$$P.Q = A +$$

$$B \times \frac{z}{1} +$$

$$C \times \frac{z}{1} \times \frac{z-1}{2} +$$

$$D \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} +$$

$$E \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} +$$

$$F \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} +$$

$$G \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} \times \frac{z-5}{6} + \&c.$$

Adeoque

Adeoque signum ipsius z mutandum est, quando $P \mathcal{Q}$ cadit ad alteras partes Ordinatae primæ, ut $p q$.

Casus Secundus.

Sit jam A_5 Ordinata in medio omnium; pone $A = B_4 + B_5$, $B = D_3 + D_4$, $C = F_2 + F_3$, $D = H + H_2$, &c. & $a = C_4$, $b = E_3$, $c = G_2$, $d = I$, &c. id est, si sint $A_6 = \alpha$, $A_7 = \beta$, $A_8 = \gamma$, $A_9 = \delta$, &c. $A_4 = \kappa$, $A_3 = \lambda$, $A_2 = \mu$, $A = \nu$, &c. Pone $A = \alpha - \kappa$, $B = \beta - 2\alpha + 2\kappa - \lambda$, $C = \gamma - 4\beta + 5\alpha - 5\kappa + 4\lambda - \mu$, $D = \delta - 6\gamma + 14\beta - 14\alpha + 14\kappa - 14\lambda + 6\mu - \nu$, &c. $a = \alpha - 2A_5 + \kappa$, $b = \beta - 4\alpha + 6A_5 - 4\kappa + \lambda$, $c = \gamma - 6\beta + 15\alpha - 20A_5 + 15\kappa - 6\lambda + \mu$, $d = \delta - 8\gamma + 28\beta - 56\alpha + 70A_5 - 56\kappa + 28\lambda - 8\mu + \nu$, &c. Et dicatur $A_5 P$, z , tum erit

$$\begin{aligned}
 P \mathcal{Q} = A_5 + \frac{Az + azz}{1.2} + \\
 \frac{2Bz + bzz}{1.2} \times \frac{zz-1}{3.4} + \\
 \frac{3Cz + czz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} + \\
 \frac{4Dz + dzz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} + \\
 \frac{5Ez + ezz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} \times \frac{zz-16}{9.10} + \\
 \&c.
 \end{aligned}$$

Casus Tertius.

Sint jam A_4 , A_5 , Ordinatae duæ in medio omnium:

$$\text{Pone } A = \frac{A_4 + A_5}{2}, B = \frac{C_3 + C_4}{2}, C = \frac{E_2 + E_3}{2}, D = \frac{G + G_2}{2}$$

10 A G + G₂

$\frac{G+G^2}{2}$, &c. $a=B_4$, $b=D_3$, $c=F_2$, $d=H$, &c.

Vel sint $A_5=\alpha$, $A_6=\beta$, $A_7=\gamma$, $A_8=\delta$, &c. $A_4=\kappa$, $A_3=\lambda$, $A_2=\mu$, $A=\nu$, &c. Deinde erunt $2A=\alpha+\kappa$, $2B=\beta-\alpha-\kappa+\lambda$, $2C=\gamma-3\beta+2\alpha+2\kappa-3\lambda+\mu$, $2D=\delta-5\gamma+9\beta-5\alpha-5\kappa+9\lambda-5\mu+\nu$, &c. Et $a=\alpha-\kappa$, $b=\beta-3\alpha+3\kappa-\lambda$, $c=\gamma-5\beta+10\alpha-10\kappa+5\lambda-\mu$, $d=\delta-7\gamma+21\beta-35\alpha+35\kappa-21\lambda+7\mu-\nu$, &c. Et sit O punctum medium inter A_4 , A_5 , atque appelletur OP , z ; eritque Ordinata

$$PQ = \frac{A+\alpha z}{4^0} + \frac{3B+bz}{4^1} \times \frac{4zz-1}{2 \cdot 3} + \frac{5C+cz}{4^2} \times \frac{4zz-1}{2 \cdot 3} \times \frac{4zz-9}{4 \cdot 5} + \frac{7D+dz}{4^3} \times \frac{4zz-1}{2 \cdot 3} \times \frac{4zz-9}{4 \cdot 5} \times \frac{4zz-25}{6 \cdot 7} + \frac{9E+ez}{4^4} \times \frac{4zz-1}{2 \cdot 3} \times \frac{4zz-9}{4 \cdot 5} \times \frac{4zz-25}{6 \cdot 7} \times \frac{4zz-49}{8 \cdot 9} + \&c.$$

In hisce duobus etiam casibus z est negativa, quando Ordinata PQ cadit ad alteras partes initii Abscissæ. Et in omnibus tribus casibus distantia communis Ordinarum ponitur unitas.

Omnes tres casus demonstrantur facillime per calculum. In casu primo pro PQ scribo successive $\alpha, \beta, \gamma, \delta, \epsilon$, &c. & pro z interea $0, 1, 2, 3, 4$, &c. quæ sunt longitudines Abscissæ ordine sequentes; & provenient æquationes

$$\alpha=A, \beta=A+B, \gamma=A+2B+C, \delta=A+3B+3C+D, \epsilon=A+4B+6C+4D+E, \&c.$$

$$\beta-\alpha$$

$$\beta - \alpha = B, \gamma - \beta = B + C, \delta - \gamma = B + 2C + D,$$

$$\varepsilon - \delta = B + 3C + 3D + E, \&c.$$

$$\gamma - 2\beta + \alpha = C, \delta - 2\gamma + \beta = C + D, \varepsilon - 2\delta + \gamma = C + 2D + E, \&c.$$

$$\delta - 3\gamma + 3\beta - \alpha = D, \varepsilon - 3\delta + 3\gamma - \beta = D + E, \&c.$$

$$\varepsilon - 4\delta + 6\gamma - 4\beta + \alpha = E, \&c.$$

Hæ *Æquationes*, capiendò earum differentias, nullo labore resolvuntur, uti videre est. Et dant eosdem ipsorum *A, B, C, D, &c.* valores, qui antea positi sunt in solutione. Et ad eundem modum demonstrantur casus duo reliqui.

Harum trium serierum unaquæque converget ad valorem *Ordinatæ P 2*, ubi *Ordinatarum* datarum differentia sunt iustæ magnitudinis. At ubi non convergunt, aliæ artes adhibendæ sunt. Sed impræsentiarum de hujus Propositionis usu pauca adjiciamus.

Designent $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \kappa, \lambda$, &c. terminos quoscunque æquidistantes, quorum differentia sunt perexiguæ; & relationes quas inter se obtinent definientur quamproxime per *Æquationes* sequentes, quæ oriuntur capiendò differentias & differentias differentiarum continuò, & ponendo eas æquales nihilo.

$$\alpha - \beta = 0$$

$$\alpha - 2\beta + \gamma = 0$$

$$\alpha - 3\beta + 3\gamma - \delta = 0$$

$$\alpha - 4\beta + 6\gamma - 4\delta + \varepsilon = 0$$

$$\alpha - 5\beta + 10\gamma - 10\delta + 5\varepsilon - \zeta = 0$$

$$\alpha - 6\beta + 15\gamma - 20\delta + 15\varepsilon - 6\zeta + \eta = 0$$

$$\alpha - 7\beta + 21\gamma - 35\delta + 35\varepsilon - 21\zeta + 7\eta - \theta = 0$$

$$\alpha - 8\beta + 28\gamma - 56\delta + 70\varepsilon - 56\zeta + 28\eta - 8\theta + \kappa = 0$$

$$\alpha - 9\beta + 36\gamma - 84\delta + 126\varepsilon - 126\zeta + 84\eta - 36\theta + 9\kappa - \lambda = 0.$$

&c.

Hæc

Hæc Tabula in usum reservanda est, ut consulatur quoties opus sit. Quod autem hæc Æquationes vel obtinent accurate, vel ad verum approximant, ubi differentiarum terminorum sunt parvæ, patet ex demonstratione casus primi Propositionis.

Assumatur quælibet Series $\frac{x}{101}, \frac{x}{102}, \frac{x}{103}, \frac{x}{104}, \frac{x}{105}, \frac{x}{106}, \&c.$ Et quærat terminus qui stat proximus ante $\frac{x}{101}$: patet quod ille est $\frac{x}{100}$; videamus ergo qualem hæc methodus exhibebit eundem. Repræsentet α terminum quæsitum, eritque

$\frac{x}{101} = \beta = 0099,0099,0099,0,$ $\frac{x}{102} = \gamma = 0098,0392,1568,7,$ $\frac{x}{103} = \delta = 0097,0873,7864,1,$ $\frac{x}{104} = \epsilon = 0096,1538,4615,4,$ $\frac{x}{105} = \zeta = 0095,2380,9523,8,$ $\frac{x}{106} = \eta = 0094,3396,2264,2.$	Æquatio. $\left\{ \begin{array}{l} 1ma \\ 2da \\ 3tia \\ 4ta \\ 5ta \\ 6ta \end{array} \right\}$	dat. $\alpha \left\{ \begin{array}{l} 0099,0099,0099,0, \\ 0099,9805,8629,3, \\ 0099,9994,3455,0, \\ 0099,9999,7824,8, \\ 0099,9999,9895,8, \\ 0099,9999,9993,1. \end{array} \right.$
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Patet ergo quod hæc methodus continue approximât: Si terminorum differentiarum fuissent minores, valores accessissent citius ad verum, & contra tardius quando differentiarum sunt majores. Hinc si in Tabulis numericis desit terminus, potest is per hanc methodum inferi.

Hoc modo etiam prodeunt ipsissimæ Series Speciosæ, quæ per alias methodos prodire solent. Proponatur $1 + zz$ Ordinata Curvæ quadrandæ: Ea est prima in serie regulari $1 + zz^{-1}, 1 + zz^0, 1 + zz^1, 1 + zz^2, 1 + zz^3, \&c.$ Ordinarum, quæ omnes præter primam dant suas areas $z, z + \frac{1}{3}z^3, z + \frac{2}{3}z^3 + \frac{1}{5}z^5, z + \frac{3}{5}z^3 + \frac{2}{7}z^5 + \frac{1}{9}z^7, \&c.$ constituentes novam seriem cujus primus terminus erit Area quæsitâ: quæ ideo invenietur ponendo pro eâ α , & pro reliquis in suo Ordine $\beta, \gamma, \delta, \epsilon, \&c.$ Prima Æquatio dat $\alpha = z$, secunda $\alpha = z - \frac{1}{3}z^3$, tertia $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5$, quarta $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7, \&c.$

&c. Est ergo universim area quæsitæ $z - \frac{1}{2}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7 + \frac{1}{9}z^9 - \frac{1}{11}z^{11}$ &c. Estque hæc Series arcus ad Tangentem z , in circulo radium habente unitati æqualem. Eam invenit *Jacobus Gregorius* noster, & cum *Collinio* communicavit initio anni 1671. à quo, mediante *Ol denburgo* ad *Leibnitium* delata est.

Sit jam &c, $e, d, c, b, a, P, \alpha, \beta, \gamma, \delta, \epsilon$, &c. Series utrinque excurrens in infinitum, ubi dantur omnes termini præter P in medio omnium. Sit $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \epsilon + e$, &c. atque erit

$$\begin{aligned}
 P &= \frac{A}{2} + \\
 &\quad \frac{A-B}{6} + \\
 &\quad \frac{5A-8B+3C}{60} + \\
 &\quad \frac{7A-14B+9C-2D}{140} + \\
 &\quad \frac{42A-96B+81C-32D+5E}{1260} + \\
 &\quad \frac{66A-165B+165C-88D+25E-3F}{2772} + \\
 &\quad \frac{429A-1144B+1287C-832D+325E-72F+7G}{24024} + \&c.
 \end{aligned}$$

Investigatur hæc Series ex Æquationibus, excerpto alternas in quibus numerus terminorum est impar. Nam earum differentiæ relinquent terminos in hac Serie; quæ itaque ad libitum produci potest.

Sit $1+z|^{-1}$ Ordinata Hyperbolæ, & quæratur Area ejus quæ jacet supra Abscissam z , quando ea evadit unitas. Hæc Ordinata est media in Serie Ordinatarum

rum, &c. $\frac{1}{1+z}^{-5}, \frac{1}{1+z}^{-4}, \frac{1}{1+z}^{-3}, \frac{1}{1+z}^{-2}, \frac{1}{1+z}^{-1},$
 $\frac{1}{1+z}^0, \frac{1}{1+z}^1, \frac{1}{1+z}^2, \frac{1}{1+z}^3$ &c. æquidistantium,
hinc inde excurrente in infinitum. Adeoque Areae ab
hisce Ordinatis genitæ constituent seriem consimilem,
cujus medius terminus erit Area quæsitæ; quæ proinde
obtinebitur per Seriem modo expositam. Quando z est
unitas, ut in casu præsentē, areae curvarum evadunt
&c. $\frac{15}{64}, \frac{7}{24}, \frac{3}{8}, \frac{1}{2},$ & $1, \frac{3}{2}, \frac{7}{3}, \frac{15}{4},$ &c. Hinc est $A = 1 +$
 $\frac{7}{2} = \frac{9}{2}, B = \frac{3}{2} + \frac{3}{8} = \frac{15}{8}, C = \frac{7}{3} + \frac{7}{24} = \frac{31}{8}, D = \frac{15}{4} + \frac{15}{64} = \frac{255}{64},$
&c. Hisce in Serie substitutis, prodit P , id est, area
Hyperbolæ, $\frac{3}{4} - \frac{3}{48} + \frac{3}{480} - \frac{3}{4480} +$ &c. id est, $\frac{3}{4} -$
 $\frac{A}{4 \cdot 3} - \frac{2B}{4 \cdot 5} - \frac{3C}{4 \cdot 7} - \frac{4D}{4 \cdot 9} - \frac{5E}{4 \cdot 11} -$ &c. Ubi jam $A,$
 $B, C, D,$ &c. more *Newtoniano*, designant terminos in suo
ordine ab initio. Calculum appono.

T E R M I N I.

Affirmativi

Negativi.

7500,0000,0000,0000,0	0625,0000,0000,0000,0
62,5000,0000,0000,0	6,6964,2857,1428,5
7440,4761,9047,6	845,5086,5800,8
97,5586,9130,8	11,3818,4731,9
1,3390,4086,1	1585,7062,8
188,7745,5	22,5708,7
2,7085,0	3260,2
393,4	47,5
5,7	7
+7563,2539,3930,7494,1	-0631,7821,3370,8041,1

Summam negativam subducens ab affirmativâ, habeo
pro Area, id est, pro Logarithmo Hyperbolico Binarii,
numerum 6931,4718,0559,9453.

Pro

Pro constructione Tabularum quarumvis numericarum percommoda est Series quæ sequitur. Designent &c. $e, d, c, b, a, \alpha, \epsilon, \gamma, \delta, \varepsilon$, &c. terminos alternos in Serie utrinque serpente in infinitum; Pone $A = \alpha + a$, $B = \epsilon + b$, $C = \gamma + c$, $D = \delta + d$, $E = \varepsilon + e$, &c. Et terminus inter α & a erit

$$\frac{A}{2} +$$

$$\frac{1}{1} \times \frac{A-B}{2^1} +$$

$$\frac{1 \cdot 3}{1 \cdot 2} \times \frac{2A-3B+C}{2^2} +$$

$$\frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \times \frac{5A-9B+5C-D}{2^3} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{14A-28B+20C-7D+E}{2^4} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{42A-90B+75C-35D+9E-F}{2^5} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \times \frac{132A-297B+275C-154D+54E-11F+G}{2^6} +$$

&c.

Hæc Series sequitur ex casu tertio Propositionis, ponendo $z=0$. Coefficientes numerales literarum sic producuntur; exempli gratiâ, in quarto termino coëfficiens literæ penultimæ C est 5; pone $5 + 1 = n$, & numeri qui proveniunt ex multiplicatione terminorum $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. erunt 1, 6, 15, 20, &c. Horum differentiæ 5, 9, 5, sunt numeri quæsitæ. Atque adeo Series ad libitum produci potest.

Datis Logarithmis numerorum 46, 48, 50, 52, 54, 56, 58 & 60; invenire Logarithmum numeri 53, qui consistit in medio omnium. Pone $l, 52 + l, 54 = A = 3,4483,9710,34$, $l, 50 + l, 56 = B = 3,4471,5803,13$.
 $l, 48$

$l, 48 + l, 58 = C = 3,4446.6923,08$, $l, 46 + l, 60 = D = 3,4409,0908,19$. Hisce valoribus in Serie scriptis, primi quatuor termini dabunt $1,7242,2586,96$ pro Logarithmo numeri 53. Et eadem ratione invenire licet quemvis alium intermedium.

In Constructione ergo Tabularum sufficit primo quærere aliquos terminos in debitis distantis. nam reliqui possunt hoc modo interferi. Etenim continuo sunt intercalandi termini primo inventi usque dum perventum fuerit ad ultimos qui desiderantur. Hoc modo habebitur tota Tabula ex datis paucis terminis sub initio pro fundamento operationis. Sed non convenit ut termini quos primo quærimus, sint omnes per totam Tabulam æquidistantes; nam si omittimus alternos ubi eorum differentia est maxima, possumus alibi per saltum omittere duos, tres, viginti aut forte plures terminos. Numerus autem terminorum inter duos datos consistentium, qui omittuntur, debet semper esse aliquis sequentium 1, 3, 7, 15, 31, 63, &c. dummodo volumus inferere eos per hanc Seriem; hoc vero neutiquam incommodabit opus.

Possunt autem pro Praxi termini in unam summam colligi, ut factum vides in hac Tabella. Prima expressio est primus terminus; secunda est summa primi & secundi; tertia est summa primi, secundi & tertii: & sic porro.

$$\begin{array}{r|l}
 2 & \frac{A}{2} \\
 4 & \frac{9A-B}{16} \\
 6 & \frac{150A-25B+3C}{256} \\
 8 & \frac{1225A-245B+49C-5D}{2048} \\
 10 & \frac{39690A-8820B+2268C-305D+35E}{65536}
 \end{array}$$

Sic

Sic datis aliquibus terminis alternis, intermedii confestim dabuntur per hæc expressiones, nullâ ratione habitâ naturæ Tabulæ particularis. Nam hæ regulæ sunt eadem in omnibus. Areæ curvarum sunt proxime æquales areis Parabolicæ figuræ quæ transit per extrema Ordinatarum suarum. Sed quoniam laboriosum nimis esset semper recurrere ad Parabolam, computavi Tabulam sequentem, quâ Areæ directæ exhibentur ex datis Ordinatis.

$$\begin{array}{r|l}
 1 & \frac{A}{1} R \\
 3 & \frac{A+4B}{6} R \\
 5 & \frac{7A+32B+12C}{90} R \\
 7 & \frac{41A+216B+27C+272D}{840} R \\
 9 & \frac{989A+5888B-928C+10496D-4540E}{28350} R \\
 11 & \frac{16067A+106300B-48525C+272400D-260550E+427368F}{598752} R.
 \end{array}$$

Hic numerus Ordinatarum est impar, A est summa primæ & ultimæ, B secundæ & penultimæ, C tertiæ & antepenultimæ; & sic porro, usque dum deventum sit ad eam in medio omnium, quæ per ultimam literam in quâque expressione repræsentatur. R est basis seu pars Abscissæ inter primam & ultimam Ordinatam interceptæ. Expressiones sunt Areæ contentæ inter Curvam, basin & Ordinatas hinc inde extremas. Tabulam pro pare numero Ordinatarum non apposui, quoniam Area cæteris paribus ex impare earum numero accuratius definitur.

Quæraturs area quæ generatur ab Ordinatâ $\overline{1 + xz}^{-1}$ & jacet supra Abscissam z quando ea evadit unitas. In

10 B
 $1 + xz$

$\overline{1 + 2z^{-1}}$, pro z scribe $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$; & prodibunt undecim Ordinatae $1, \frac{100}{101}, \frac{25}{101}, \frac{100}{109}, \frac{25}{109}, \frac{4}{5}, \frac{25}{34}, \frac{100}{149}, \frac{25}{41}, \frac{100}{281}, \frac{1}{2}$. Hinc est $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{100}{101} + \frac{100}{181} = \frac{28100}{18281}$, $C = \frac{25}{25} + \frac{25}{41} = \frac{1675}{1066}$, $D = \frac{100}{109} + \frac{100}{149} = \frac{25800}{16241}$, $E = \frac{25}{29} + \frac{25}{34} = \frac{1575}{986}$, $F = \frac{4}{5}$. Hisce valoribus substitutis in ultimâ expressione, & unitate pro R , invenies aream esse 785398187. Justus est hic numerus in septimâ figurâ, in octavâ verum superans Binario.

Si undecim Ordinatae non dent aream satis exactam, erige plures; & concipe aream divisam esse in plures partes, quarum quamque seorsum quærens habebis pro lubitu justam.

Valor ipsius $\overline{1 + Q^n}$ exprimi potest per quaecunque trium serierum sequentium.

$$\begin{aligned} \overline{1 + Q^n} &= 1 + \\ &Q \times \frac{n}{1} + \\ &Q^2 \times \frac{n}{1} \times \frac{n-1}{2} + \\ &Q^3 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} + \\ &Q^4 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} + \\ &Q^5 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} + \&c. \end{aligned}$$

$$\begin{aligned} \text{Vel } \overline{1 + Q^n} &= 1 + \\ &R \times \frac{n}{1} + \\ &R^2 \times \frac{n}{1} \times \frac{n+1}{2} + \\ &R^3 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} + \end{aligned}$$

$$R^4 \times$$

$$R^4 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} +$$

$$R^5 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} \times \frac{n+4}{5} + \&c.$$

posito scilicet $R = \frac{1+Q}{Q}$. Vel

$$\overline{1+Q}^n = 1 +$$

$$\frac{2+n+1 \times Q}{1+Q^1} \times Q \times \frac{n}{1.2} +$$

$$\frac{4+n+2 \times Q}{1+Q^2} \times Q^2 \times \frac{n}{1.2} \times \frac{n-1}{3.4} +$$

$$\frac{6+n+3 \times Q}{1+Q^3} \times Q^3 \times \frac{n}{1.2} \times \frac{n-1}{3.4} \times \frac{n-4}{5.6} +$$

$$\frac{8+n+4 \times Q}{1+Q^4} \times Q^4 \times \frac{n}{1.2} \times \frac{n-1}{3.4} \times \frac{n-4}{5.6} \times \frac{n-9}{7.8} +$$

$$\frac{10+n+5 \times Q}{1+Q^5} \times Q^5 \times \frac{n}{1.2} \times \frac{n-1}{3.4} \times \frac{n-4}{5.6} \times \frac{n-9}{7.8} \times \frac{n-16}{9.10}$$

+ &c.

Primæ duæ Series demonstrantur per Casum primum Propositionis. Nam si $\overline{1+Q}^0$, $\overline{1+Q}^1$, $\overline{1+Q}^2$, $\overline{1+Q}^3$, $\overline{1+Q}^4$, &c. designent Ordinatas totidem æquidistantes in Parabolicâ figurâ. erit $\overline{1+Q}^n$ ejusdem Ordinata, cujus distantia à $\overline{1+Q}^0$ est n . Et sic prodit Series prima. At si in alia Parabola $\overline{1+Q}^0$, $\overline{1+Q}^{-1}$, $\overline{1+Q}^{-2}$, $\overline{1+Q}^{-3}$, $\overline{1+Q}^{-4}$, &c. sint æquidistantes Ordinatæ, erit $\overline{1+Q}^n$ Ordinata in eadem, cujus distantia à $\overline{1+Q}^0$ est $-n$; sic proveniet Series secunda. Sit jam in tertia Parabola &c. $\overline{1+Q}^{-4}$, $\overline{1+Q}^{-3}$, $\overline{1+Q}^{-2}$, $\overline{1+Q}^{-1}$, $\overline{1+Q}^0$, $\overline{1+Q}^1$, $\overline{1+Q}^2$, $\overline{1+Q}^3$, $\overline{1+Q}^4$, &c. Series Ordinarum
10 B 2 æqui-

æquidistantium hinc inde progrediens in infinitum, eritque in eadem $\overline{1 + 2^n}$ Ordinata, distantia n à termino medio $\overline{1 + 2^0}$ remota. Et sic provenit Series tertia per Casum Secundum Propositionis. Prima abruptit quando est n integer & affirmativus, secunda quando est n integer & negativus, & tertia in casu utroque abruptit. Per harum quamque radices numerales commodè evolvuntur in Series. Tertia reliquis multo citius convergit: ejus terminus secundus adhiberi potest pro correctione, ubi fit extractio per repetitionem calculi.

Halleius in sua methodo construendi Logarithmos, ex prima harum serierum demonstrat Seriem *Mercatoris* pro Quadratura Hyperbolæ. Sit ejus Ordinata $\overline{1 + z}^{-1}$, vel $\overline{1 + z}^{n-1}$, existente n numero infinite parvo; unde per methodos Quadrandi, area quæ jacet supra Abscissam z , id est, Logarithmus numeri $\overline{1 + z}$, erit $\frac{1 - \overline{1 + z}^n}{n}$. Est vero per primam Seriem $\overline{1 + z}^n = 1 + \frac{n}{1} z + \frac{n}{1} \times \frac{n-1}{2} z^2 + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} z^3 + \&c.$ adeoque in casu præsentè, ubi est n infinite parvus, est $\overline{1 + z}^n = 1 + \frac{n}{1} z - \frac{n}{2} z^2 + \frac{n}{3} z^3 - \frac{n}{4} z^4 + \&c.$ quo substituto in valore areæ, ea prodit $z - \frac{1}{2} z^2 + \frac{1}{3} z^3 - \frac{1}{4} z^4 + \frac{1}{5} z^5 - \&c.$ quæ est Series *Mercatoris*.

Similiter per Seriem secundam prodit hæc regula; Sit datus numerus $\overline{1 + z}$, pone $R = \frac{z}{1 + z}$, eritque ejus Logarithmus $R + \frac{1}{2} R^2 + \frac{1}{3} R^3 + \frac{1}{4} R^4 + \frac{1}{5} R^5 + \&c.$

Per Seriem tertiam provenit sequens regula. Sit quilibet numerus R , pone $z = \frac{R-1}{2R}$, eritque ejus Logarithmus

garithmus $\frac{RR-1}{2R} = \frac{1}{2} Az - \frac{2}{3} Bz - \frac{2}{7} Cz - \frac{4}{9} Dz - \frac{1}{11} Ez$
 — &c. Ubi A, B, C, D, E , &c. more *Newtoniano* designant terminos Seriei sicut ab initio. Hæc Series, ut ea ex qua deducitur, reliquis duabus multis vicibus celebrius approximatur: estque eadem generalius expressa quam, ex fundamento haud absimili, pro inventione Logarithmi Binarii prius dedimus.

Methodus inveniendi valores Serierum Arithmeticarum utcumque tarde convergentium.

In aliquibus Seriebus summa terminorum haberi nequit nisi ad paucissima figurarum loca, dummodo præter simplicem eorum additionem aliæ artes non adhibeantur. Proponatur jam Series quælibet cujus termini omnes iisdem signis afficiuntur, & quorum proximi continue tendunt esse inter se æquales; quales sunt sequentes $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \frac{1}{7 \cdot 8} + \&c.$ $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \&c.$ Collige summam aliquot terminorum sub initio, ii proxime addendi sint $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$, &c. In numeris proximis sit $r = \frac{\alpha\gamma - \beta\beta}{\alpha\beta - 2\alpha\gamma + \beta\gamma}$, & quantitatum $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, $\alpha + \beta \times \frac{\beta + r\gamma}{\beta - \gamma}$, $\alpha + \beta + \gamma \times \frac{\gamma + r\delta}{\gamma - \delta}$, $\alpha + \beta + \gamma + \delta \times \frac{\delta + r\epsilon}{\delta - \epsilon}$, $\alpha + \beta + \gamma + \delta + \epsilon \times \frac{\epsilon + r\zeta}{\epsilon - \zeta}$, &c. differentiarum sint a, b, c, d, e , &c. Deinde in numeris proximis sit $s = \frac{ac - bb}{ab - 2ac + bc}$, & ipsorum $a \times \frac{a + sb}{a - b}$, $a + b \times \frac{b + sc}{b - c}$, $a + b + c \times \frac{c + sd}{c - d}$, $a + b + c + d \times \frac{d + se}{d - e}$, &c. differentiarum sint A, B, C, D , &c. & sit $t = \frac{AC - BB}{AB - 2AC + BC}$: atque sic procede

cede quoad libuerit. Tum erit $\alpha + \beta + \gamma + \delta + \epsilon + \&c. = \alpha \times \frac{a+r\beta}{a-\beta} + \alpha \times \frac{a+sb}{a-b} + A \times \frac{A+B}{A-B} + \&c.$ atque ultra duos primos terminos hujus novæ Seriei raro opus erit progredi.

Ut si desideretur valor Seriei $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} + \&c.$ collige primos 21 terminos, quorum summam reperio fore 6813,8410,1885. Termini proxime addendi sunt $\alpha = ,0005,2854,1226$, $\beta = ,0004,8309,1787$, $\gamma = ,0004,4326,2411$, $\delta = ,0004,0816,3265$, &c. Hinc fit $r = 1$ proxime, & $\alpha \times \frac{a+r\beta}{a-\beta} = ,0117,6449,6282$, $a = -,0000,0017,5096$, $b = -,0000,0014,7410$, $c = -,0000,0012,4986$, &c. Unde $s = \frac{1}{2}$ prope, & $\alpha \times \frac{a+sb}{a-b} = -,0000,0141,8111$, quem propter signum negativum subduco ab $\alpha \times \frac{a+r\beta}{a-\beta}$, & remanet, 0117,6307, 8171: hic additus summæ primo inventæ 6813,8410, 1885, dat pro summa totius Seriei numerum 6931, 4718,0056, qui justus est in nonâ decimali; at ante duas hæc correctiones summa erat justa in primâ figura solâ. Si animus sit propius scopum attingere, pergendum erit ad approximationes sequentes. Si termini Seriei diversa habeant signa, conjungendi sunt, ut omnes eadem tandem habeant, ut in Serie $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \&c.$ conjunctis terminis ea evadit $\frac{2}{1.3} + \frac{2}{3.7} + \frac{2}{9.11} + \frac{2}{13.15} + \&c.$ Sed hic notandum est quod differentiæ a, b, c, d, e , &c. ut & A, B, C, D , &c. colligi debent subducendo quantitates antecedentes de subsequen-
tibus. Et in omnibus hujusmodi Seriebus si p, q, r , repræsentent tres terminos ordine sequentes, p pri-
mum,

num, q secundum, r tertium, & rectangulum $\frac{p+r}{2} \times q$ non sit majus pr , valor Seriei erit infinite magnus: at magnitudinis semper finitæ ubi accidit contrarium. Potest hæc regula nonnunquam fallere, ubi termini p , q , r parum distant ab initio Seriei, at si consistent inter eos ab initio aliquantum remotos, evadet regula certissima.

Ad alia Serierum genera debent aliæ regulæ adhiberi. Sit Series regularium Polygonorum Circulo Inscriptorum, existente Radio unitate.

$$\begin{aligned} H &= 2,0000,0000,0000,000 | 4 \\ G &= 2,8284,2712,4746,190 | 8 \\ F &= 3,0614,6745,8920,718 | 16 \\ E &= 3,1214,4515,2258,051 | 32 \\ D &= 3,1365,4849,0545,938 | 64 \\ C &= 3,1403,3115,6954,752 | 128 \\ B &= 3,1412,7725,0932,772 | 256 \\ A &= 3,1415,1380,1144,299 | 512 \end{aligned}$$

Dicatur jam ultimum Polygonum A , penultimum B , antepenultimum C , & reliqua in suo ordine retrorsum D , E , F , &c. atque area Circuli quæsitæ erit $A + \frac{A-B}{3}$

$$+ \frac{4A-5B+C}{3 \cdot 15} + \frac{64A-84B+21C-D}{3 \cdot 15 \cdot 63} +$$

$$\frac{4096A-5440B+1428C-85D+E}{3 \cdot 15 \cdot 63 \cdot 255} + \&c. \quad \text{Ubi si pro } A,$$

B , C , D , E , &c. scribantur proprii valores, primi quatuor termini dabunt 3,1415,9265,3589,790 pro area Circuli. Hæc autem Series est generalis, ex natura Circuli neutiquam dependens: applicabilis est quotiescunque numerorum approximantium differentię priores sunt posteriorum quali quadruplæ. Factores in Denominatoribus sunt dignitates integræ numeri 4 unitatibus mi-

nutæ :

aturæ: quibus datis, coefficientes literarum in diversis terminis formantur ex multiplicatione continua numerorum 1. $\frac{n}{3}$, $\frac{n-3}{15}$, $\frac{n-15}{63}$, $\frac{n-63}{255}$, &c. Ubi pro n substituendus est ultimus Factorum in Denominatore.

Ultima quantitatum $x - 1$, $2\sqrt{x} - 2$, $4\sqrt[3]{x} - 4$, $8\sqrt[4]{x} - 8$, $16\sqrt[5]{x} - 16$, &c. æqualis est Logarithmo numeri x . Pro x scribe 2, & per repetitam extractionem radicis quadratæ exhibunt numeri

$$M = 1,0000,0000,0000,0000.$$

$$L = 8284,2712,4746,1901.$$

$$I = 7568,2864,0010,8843.$$

$$H = 7240,6186,1322,0613.$$

$$G = 7083,8051,8838,6214.$$

$$F = 7007,0875,6931,7337.$$

$$E = 6969,1430,7308,8294.$$

$$D = 6950,2734,2438,7611.$$

$$C = 6940,8641,2851,8363.$$

$$B = 6936,1658,4759,4014.$$

$$A = 6933,8182,9699,9493.$$

Dicatur ultimus numerorum A , penultimus B , & sic retro, atque Logarithmus quæsitus erit $A + \frac{A-B}{1} + \frac{2A-3B+C}{1 \cdot 3} + \frac{8A-14B+7C-D}{1 \cdot 3 \cdot 7} + \frac{64A-120B+70C-15D+E}{1 \cdot 3 \cdot 7 \cdot 15} + \&c.$ Primi quinque termini dant 6931,4718,0559, 9457 pro Logarithmo Hyperbolico Binarii. Et quomodo hæc Series procedit in infinitum facile colligitur ex eo quod de priore diximus: estque etiam universalis, proprietates Hyperbolæ minime respiciens.

Extenditur quoque Methodus hæcce Differentialis ad Resolutionem \mathcal{A} equationum & alia quamplurima quorum hic non fit mentio. Continetque fundamenta Serierum generalissima; ut in Reductione \mathcal{A} equationum Irrationalium & Fluxionalium brevi forsan monstrabo.